List of Talks

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Silviu Blnescu, University Politehnica Bucharest:

On the arithmetic quasi-depth

Let $h : \mathbb{Z} \to \mathbb{Z}_{\geq 0}$ be a nonzero function with $h(k) = 0$ for $k \ll 0$. We define the *quasi depth* of h by

$$\operatorname{qdepth}(h) = \max\{d \ : \ \sum_{j \leq k} (-1)^{k-j} \binom{d-j}{k-j} h(j) \geq 0 \text{ for all } k \leq d\}.$$

We show that $\operatorname{qdepth}(h)$ is a natural generalization for the quasi depth of a subposet $\mathrm{P} \subset 2^{[n]}$ and we prove some basic properties of it.

Given $h(j) = \begin{cases} P(j), & j \geq 0 \\ 0, & j < 0 \end{cases}$, where $P(j)$ is a polynomial function of degree n with $P(0) > 0$, we prove that $\operatorname{qdepth}(h) \leq 2^{n+1}$. Moreover, if $h(j) = \begin{cases} aj^n + b, & j \geq 0 \\ 0, & j < 0 \end{cases}$, where a, b are positive integers, we give precise formulae for $\operatorname{qdepth}(h)$ when $n = 1, 2$ and we conjecture a formula, in general.

We consider $h_2(j) = \begin{cases} aj^2 + bj + e, & j \geq 0 \\ 0, & j < 0 \end{cases}$, and $h_3(j) = \begin{cases} aj^3 + bj^2 + cj + e, & j \geq 0 \\ 0, & j < 0 \end{cases}$, where a, b, c, e are some integers with $a, e > 0$ such that h_2 and h_3 take only nonnegative values. We prove that if $b < 0$ and $b^2 \leq 4ae$ then $\operatorname{qdepth}(h_2) \leq 11$, and, if $b < 0$ and $b^2 > 4a3$ then $\operatorname{qdepth}(h_2) \leq 13$. Also, we show that if $b < 0$ and $b^2 \leq 3ac$ then $\operatorname{qdepth}(h_3) \leq 67$.

References

[1] S. Bălănescu, M. Cimpoeaş, *On the arithmetic quasi depth*, arXiv:2309.10521 (2023).

[2] S. Bălănescu, M. Cimpoeaş, *On the quasi depth of quadratic and cubic functions*, arXiv:2402.01478 (2024).

Nicolae Bonciocat, IMAR:

Resultants, prime numbers and irreducible polynomials

We present several applications of the resultant (the determinant of the Sylvester matrix) in factorization problems for univariate and multivariate polynomials. Such factorization problems include finding upper bounds for the number of irreducible factors (in particular irreducibility criteria), or for the multiplicities of the irreducible factors (in particular separability criteria). The multivariate case requires a non-Archimedean setting.

Mihai Cipu, IMAR:
Explicit formulas for the solutions of simultaneous Pell equations

It is well known that systems of Pell equations have at most two solutions in positive integers. Moreover, these solutions appear in linear second-order recurrent sequences. For many classes of coefficients, more precise information is available. For instance, for any integers a, b, if b is odd and its square-free part has at most two prime divisors, then the simultaneous Pell equations $x^2 - (a^2-1)y^2 = 1$, $y^2 - bz^2 = 1$ have a common solution if and only if b divides $4a^2 - 1$ and the quotient is a perfect square. When it exists, the solution is unique and of the form

$$(x, y, z) = \left(2a^2 - 1, 2a, \sqrt{(4a^2-1)/b}\right).$$

In this talk, various extensions of this result are presented.

Vlad Matei, IMAR:
Some new families with prescribed Galois group

In this talk we will explore some new families of polynomials with rational coefficients and with prescribed Galois group in degrees 3 and 4.

Mihai Prunescu, University of Bucharest and IMAR:
Closed term formulas for tau, sigma, phi and other functions

The number of divisors of n is computed by performing a fixed number of arithmetical operations carried in a given order, by evaluating an arithmetic closed term with exponentiation that depends on the argument n only. This can be also done for the sum of the divisors, for Euler's totient function, and for many other number-theoretical functions.

Lorenzo Sauras-Altuzarra, Technische Universitt Wien:
Proof mining in Fermat numbers

In his article ”Note on the generalization of calculations”, Matthias Baaz introduced an algorithm from extractive proof theory and, as an application, he obtained a new property of the factors of Fermat numbers. Around two decades later, I showed, in my article ”Some applications of Baaz's generalization method to the study of the factors of Fermat numbers”, a geometric characterization of such factors. This characterization generalized Baaz's pioneer result and was accompanied by a very general conjecture involving point-lattices. After presenting the aforementioned conjecture in several conferences, Ren Schoof and Mabud Sarkar informed me about their partial answers. In addition, Vasile Brnznescu observed a relation with the theory of unimodular matrices which led to a joint result with Gergely Harcos; Daniele Parisse found an important connection with the theory of generalized Pillai equations, which I continued developing; and Robert Tichy posed a few concrete questions which might approach us to its resolution. In this presentation I will speak about these theorems and open problems, some of which have just appeared in my doctoral dissertation and other ones are yet to be published.